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Global Nonexistence of Smooth Electric Induction Fields

in

One-Dimensional Nonlinear Dielectrics

by

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ABSTRACT

Coupled nonlinear wave equations are derived for the evolution of the components of the electric induction field D in a class of rigid nonlinear dielectrics governed by the nonlinear constitutive relation $E = \lambda(D)D$, where E is the electric field and $\lambda \geq 0$ is a scalar-valued vector function. For the special case of a finite one-dimensional dielectric rod, embedded in a perfect conductor, and subjected to an applied electric field, which is perpendicular to the axis of the rod, and depends only on variations of the coordinate along that axis, it is shown that, under relatively mild conditions on λ , solutions of the corresponding initial-boundary value problem for the electric induction field can not exist globally in time in the L^2 sense; under slightly stronger assumptions on the constitutive function λ , a standard Riemann Invariant argument may be applied to show that the space-time gradient of the non-zero component of the electric induction field must blow-up in finite time. Some growth estimates for solutions, which are valid on the maximal time-interval of existence are also derived.

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1. Evolution Equations for a Class of Nonlinear Dielectrics

Theories of material dielectric behavior are based upon a set of field equations (Maxwell's equations) and a set of constitutive relations which hold among the electromagnetic field vectors. In a Lorentz reference frame (x^i, t), $i = 1, 2, 3$, where the (x^i) represent rectangular Cartesian coordinates, and t is the time parameter, the local forms of Maxwell's equations are given by

$$(1.1) \quad \left\{ \begin{array}{l} -\frac{\partial \mathbf{B}}{\partial t} + \operatorname{curl} \mathbf{E} = \mathbf{0}, \quad \operatorname{div} \mathbf{B} = 0, \\ \operatorname{curl} \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{0}, \quad \operatorname{div} \mathbf{D} = 0, \end{array} \right.$$

provided that the density of free current, the magnetization, and the density of free charge all vanish. In (1.1), \underline{B} , \underline{E} , and \underline{H} are, respectively, the magnetic flux density, electric field, and magnetic intensity while $\underline{D} \equiv \epsilon_0 \underline{E} + \underline{P}(\underline{E})$, $\epsilon_0 > 0$ a physical constant and \underline{P} the polarization vector, is the electric induction field; the relations (1.1) hold in some bounded open domain $\Omega \subseteq \mathbb{R}^3$ which is filled with a rigid, nonconducting, dielectric substance. The precise nature of the dielectric medium in Ω is determined by specifying a set of constitutive equations relating \underline{E} , \underline{D} , \underline{H} , and \underline{B} ; indeed, without the specification of additional relations among the electromagnetic field vectors, the set of equations (1.1) represents an indeterminate system.

There is, in existence, a wide variety of constitutive hypotheses which have been associated with theories of nonconducting, rigid, dielectric media; the simplest of these is that associated with the dielectric response of a vacuum in which there hold the classical constitutive relations

$$\mathfrak{D} = \epsilon_0 \mathfrak{E}, \quad \mathfrak{H} = \mu_0^{-1} \mathfrak{E},$$

where the fundamental physical constants ϵ_0, μ_0 satisfy $\epsilon_0 \mu_0 = c^{-2}$, c being the speed of light in a vacuum. In 1873 Maxwell [1] proposed as a set of constitutive laws for a linear, rigid, stationary non-conducting dielectric the relations

$$\underline{D} = \underline{\epsilon} \cdot \underline{E}, \quad \underline{B} = \underline{\mu} \cdot \underline{H}$$

where $\underline{\epsilon}, \underline{\mu}$ are constant second-order tensors which are proportional to the identity tensor if the material is isotropic. A set of constitutive relations, which are still linear, but which take into account certain memory effects in the dielectric, were proposed by Maxwell in 1877 and subsequently used by Hopkinson [2] in connection with his studies on the residual charge of the Leyden jar; the Maxwell-Hopkinson dielectric is governed by the set of constitutive relations ($x \in \Omega$):

$$(1.2) \quad \begin{cases} \underline{D}(x,t) = \epsilon \underline{E}(x,t) + \int_{-\infty}^t \phi(t-\tau) \underline{E}(x,\tau)d\tau \\ \underline{H} = \mu^{-1} \underline{B} \end{cases}$$

where $\epsilon > 0, \mu > 0$ and $\phi(t), t \geq 0$ is a continuous monotonically decreasing function of t , $0 \leq t < \infty$. Noting that the Maxwell-Hopkinson constitutive relations do not account for the observed absorption and dispersion of electromagnetic waves in material non-conductors, Toupin and Rivlin [3] generalized the constitutive relations (1.2) and introduced the concepts of holohedral isotropic and hemihedral dielectric response; while the response incorporated into both of these theories is linear, they are more sophisticated than (1.2) in the sense that magnetic memory effects and coupling of electric and magnetic effects is built into the constitutive theory. The qualitative behavior of the electric induction field in a rigid non-conducting dielectric exhibiting holohedral isotropic response has been studied by this author in a series of recent papers [4] - [6].

In this paper we will be concerned with initial-boundary value problems associated with the evolution of the components of the electric induction field D in a relatively simple class of materials exhibiting nonlinear dielectric response. A rather general theory of nonlinear dielectric behavior which allows for both electric and magnetic memory effects, but still effects an a priori separation of electric and magnetic response, was proposed by Volterra [7] in 1912 in the form of the constitutive relations

$$(1.3) \quad \left\{ \begin{array}{l} D(x,t) = \xi \cdot E(x,t) + \int_{-\infty}^t \mathcal{D}(E(x,\tau)), \quad x \in \Omega \\ B(x,t) = \mu \cdot H(x,t) + \int_{-\infty}^t \mathcal{B}(H(x,\tau)), \quad x \in \Omega \end{array} \right.$$

The constitutive relations (1.3) reduce to those considered in [2], [3] under special assumptions relative to the functionals \mathcal{D} , \mathcal{B} , i.e., if $\mathcal{B} = 0$, \mathcal{D} is linear and isotropic, and $\xi = \epsilon I$, $\mu = \mu I$, then (1.3) is easily seen to reduce to (1.2); the particular class of nonlinear dielectrics to be considered in this exposition results by specializing (1.3) to the situation where $\mu = \mu I$, $\mu > 0$, $\mathcal{B} = 0$, and electric field memory effects are negligible, i.e.,

$$(1.4a) \quad \left\{ \begin{array}{l} D(x,t) = \mathcal{D}E(x,t), \quad x \in \Omega \\ B(x,t) = \mu H(x,t), \quad x \in \Omega \end{array} \right.$$

We shall further assume that $\det \left[\frac{\partial \mathcal{D}_{ij}}{\partial E_j} \right]_{E=0} \neq 0$, so that in a (Euclidean) neighborhood of $E = 0$, the relations (1.4a) may be inverted so as to yield the constitutive equations

$$(1.4b) \quad \left\{ \begin{array}{l} E(x,t) = \mathcal{E}(D(x,t)), \quad x \in \Omega \\ H(x,t) = \mu^{-1}B(x,t), \quad x \in \Omega \end{array} \right.$$

As the vector function \underline{E} is still completely arbitrary, the constitutive theory defined by (1.4b) is still far too general to provide a tractable system of evolution equations for the electromagnetic field in Ω ; we will, therefore, confine our attention to that special case of (1.4b) for which there exists a scalar-valued vector function $\lambda(\xi)$ such that $\underline{E}(\xi) = \lambda(\xi)\xi, \forall \xi$ with real components ξ_i . Thus, the final form of the constitutive relations which define the nonlinear dielectric response to be considered here is given by

$$(1.5) \quad \left\{ \begin{array}{l} \underline{E}(\underline{x}, t) = \lambda(\underline{D}(\underline{x}, t)) \underline{D}(\underline{x}, t), \quad \underline{x} \in \Omega \\ \underline{H}(\underline{x}, t) = \mu^{-1} \underline{B}(\underline{x}, t), \quad \underline{x} \in \Omega \quad (\mu > 0) \end{array} \right.$$

For now we will simply assume that $0 \leq \lambda(\xi) < \infty, \forall \xi$, with $\lambda(\xi) > 0, \forall \xi \neq 0$; further assumptions on the constitutive function λ will be imposed below. It seems worthwhile to note, in passing, that electromagnetic constitutive relations of the form (1.5) or, to be somewhat more accurate, the inverted relations

$$(1.6) \quad \left\{ \begin{array}{l} \underline{D}(\underline{x}, t) = \chi(\underline{E}(\underline{x}, t)) \underline{E}(\underline{x}, t), \quad \underline{x} \in \Omega \\ \underline{B}(\underline{x}, t) = \mu \underline{H}(\underline{x}, t), \quad \underline{x} \in \Omega \quad (\mu > 0) \end{array} \right.$$

have appeared in the recent literature; e.g., Rivlin [8] considers (1.6) and indicates that in an isotropic material conforming to this constitutive hypothesis the dielectric "constant" χ must be an even function of the magnitude of \underline{E} , i.e., $\chi = \chi(|\underline{E}| \cdot |\underline{E}|)$. However, there does not seem to exist, anywhere in the literature, equations for the evolution of the components of either the electric or electric induction fields in a dielectric exhibiting nonlinear response; for the simple nonlinear dielectric which is governed by the constitutive hypothesis (1.5) such a system of evolution equations is given by the following

Lemma 1. Let $\Omega \subseteq \mathbb{R}^3$ be a bounded domain and assume that Ω is filled with a rigid, nonlinear, nonconducting dielectric substance which conforms to the constitutive hypothesis (1.5). Then, in Ω , the components $D_i(x, t)$, of the electric induction field, satisfy the coupled system of nonlinear wave equations

$$(1.7) \quad \mu \frac{\partial^2 D_i}{\partial t^2} = \nabla^2 (\lambda(D) D_i) - \frac{\partial}{\partial x_i} (\text{grad } \lambda(D) \cdot D), \quad i = 1, 2, 3.$$

Proof. We begin with the identity

$$\Delta A = \text{grad}(\text{div } A) - \text{curl curl } A$$

which is valid for any sufficiently smooth vector field on Ω ; applied to the electric field (\cdot, t) the identity yields

$$(1.8) \quad \nabla^2 E_i = \frac{\partial}{\partial x_i} (\text{div } E) - (\text{curl curl } E)_i; \quad i = 1, 2, 3.$$

In view of Maxwell's equations (1.1), and the second constitutive relation in (1.5), we have

$$\begin{aligned} \text{curl curl } E &= - \text{curl} \left(\frac{\partial B}{\partial t} \right) \\ &= - \mu \text{curl} \left(\frac{\partial H}{\partial t} \right) \\ &= - \mu \frac{\partial}{\partial t} (\text{curl } H) \\ &= - \mu \frac{\partial^2 D}{\partial t^2}, \end{aligned}$$

so that (1.8) has the equivalent form

$$(1.9) \quad \mu \frac{\partial^2 D_i}{\partial t^2} = \nabla^2 E_i - \frac{\partial}{\partial x_i} (\text{div } E), \quad i = 1, 2, 3$$

By (1.4b),

$$\begin{cases} \text{div } E = \frac{\partial E_j}{\partial x_k} \cdot \frac{\partial D_k}{\partial x_j} = A_{jk}(D) \frac{\partial D_k}{\partial x_j} \\ \nabla^2 E_i = \frac{\partial}{\partial x_k} \left(\frac{\partial E_i}{\partial x_k} \right) = \frac{\partial}{\partial x_k} (A_{il}(D) \frac{\partial D_k}{\partial x_l}), \end{cases}$$

where $A_{ij}(D) \equiv \frac{\partial E_i}{\partial D_j}$, and the standard summation convention has been employed.

Thus (1.9) becomes

$$(1.10) \quad \mu \frac{\partial^2 D_i}{\partial t^2} = \frac{\partial}{\partial x_k} (A_{il}(D) \frac{\partial D_l}{\partial x_k}) - \frac{\partial}{\partial x_i} (A_{jk}(D) \frac{\partial D_k}{\partial x_j})$$

However, by virtue of our hypothesis that $E_i(D) = \lambda(D) D_i$, we easily find that

$$A_{ij}(D) = \lambda(D) \delta_{ij} + \frac{\partial \lambda}{\partial D_j} D_i$$

and therefore

$$(1.11) \quad \mu \frac{\partial^2 D_i}{\partial t^2} = \frac{\partial}{\partial x_k} ([\lambda(D) \delta_{il} + \frac{\partial \lambda}{\partial D_l} D_i] \frac{\partial D_l}{\partial x_k}) - \frac{\partial}{\partial x_i} ([\lambda(D) \delta_{jk} + \frac{\partial \lambda}{\partial D_k} D_j] \frac{\partial D_k}{\partial x_j})$$

where we sum on each repeated index; expanding (1.11) and using the Maxwell relation $\operatorname{div} D = \frac{\partial D_i}{\partial x_i} = 0$, we obtain the stated result (1.7), i.e.,

$$(1.12) \quad \mu \frac{\partial^2 D_i}{\partial t^2} = \frac{\partial}{\partial x_k} (\lambda(D) \frac{\partial D_i}{\partial x_k}) + \frac{\partial}{\partial x_k} (\frac{\partial \lambda}{\partial x_k} D_i) - \frac{\partial}{\partial x_i} (\frac{\partial \lambda}{\partial x_j} D_j) \\ = \frac{\partial^2}{\partial x_k \partial x_k} (\lambda(D) D_i) - \frac{\partial}{\partial x_i} (\frac{\partial \lambda}{\partial x_j} D_j)$$

Q.E.D.

We now assume that $\partial\Omega$ is sufficiently smooth to admit of applications of the divergence theorem and we denote by $\nu(x)$ the exterior unit normal to $\partial\Omega$ at a point $x \in \partial\Omega$; we also denote by $t(x)$ a generic vector in the tangent plane to $\partial\Omega$ at $x \in \partial\Omega$. The evolution equations (1.7) are to hold in some cylinder $\Omega \times [0, T]$, $T > 0$, in \mathbb{R}^4 and we now associate with this system a set of initial and boundary data. In Ω we require that

$$(1.13) \quad D_i(x,0) = f_i(x), \quad \frac{\partial D_i}{\partial t}(x,0) = g_i(x), \quad \begin{cases} x \in \Omega \\ i=1,2,3 \end{cases}$$

while standard results from electromagnetic theory [9,§13] dictate that

$$(1.14a) \quad [D(x,t) \cdot \underline{v}(x)] = \sigma(x), \quad (x,t) \in \partial\Omega \times [0,T]$$

$$(1.14b) \quad [\underline{E}(x,t) \cdot \underline{t}(x)] = 0, \quad (x,t) \in \partial\Omega \times [0,T]$$

In the set of relations (1.14), $[F(x)]$ denotes the jump of the scalar-valued function F across $\partial\Omega$ at $x \in \partial\Omega$ while $\sigma(x)$ denotes the density of surface charge at the point $x \in \partial\Omega$; these boundary conditions can be written in an alternative form as follows: If we let $\underline{D}^*(x,t)$ denote the electric induction field at points $(x,t) \in \mathbb{R}^3/\Omega \times [0,T]$ then (1.14a), (1.14b) are clearly equivalent to

$$(1.15a) \quad \underline{D}(x,t) \cdot \underline{v}(x) - \underline{D}^*(x,t) \cdot \underline{v}(x) = \sigma(x), \quad (x,t) \in \partial\Omega \times [0,T]$$

$$(1.15b) \quad \lambda(\underline{D}(x,t)) \underline{D}(x,t) \cdot \underline{t}(x) = \underline{E}^*(x,t) \cdot \underline{t}(x), \quad (x,t) \in \partial\Omega \times [0,T]$$

where $\underline{E}^*(x,t)$, $(x,t) \in \mathbb{R}^3/\Omega \times [0,T]$, is the electric field associated with $\underline{D}^*(x,t)$. In particular, if $\Omega \subset \tilde{\Omega} \subseteq \mathbb{R}^3$, and $\tilde{\Omega}/\Omega$ is filled with a perfect conductor (in which $\underline{D}^* = \underline{E}^* = \underline{0}$) then (1.15a), (1.15b) reduce to

$$(1.16a) \quad \underline{D}(x,t) \cdot \underline{v}(x) = \sigma(x), \quad (x,t) \in \partial\Omega \times [0,T]$$

$$(1.16b) \quad \lambda(\underline{D}(x,t)) \underline{D}(x,t) \cdot \underline{t}(x) = 0, \quad (x,t) \in \partial\Omega \times [0,T].$$

In this paper we wish to consider that particular subcase of the general initial-boundary value problem (1.7), (1.13), (1.16a,b) which corresponds to the assumption that the geometry of Ω is one-dimensional (non-linear dielectric

rod) and that the rod is subjected to an applied electric field, which is perpendicular to the axis of the rod, and depends only on variations of the coordinate along that axis; corresponding to the appropriate specialization of the boundary conditions (1.16a,b) which result in the physical assumption that the rod is embedded in a perfect conductor. We assume, therefore, that the rod occupies the configuration depicted in Figure 1. (below):

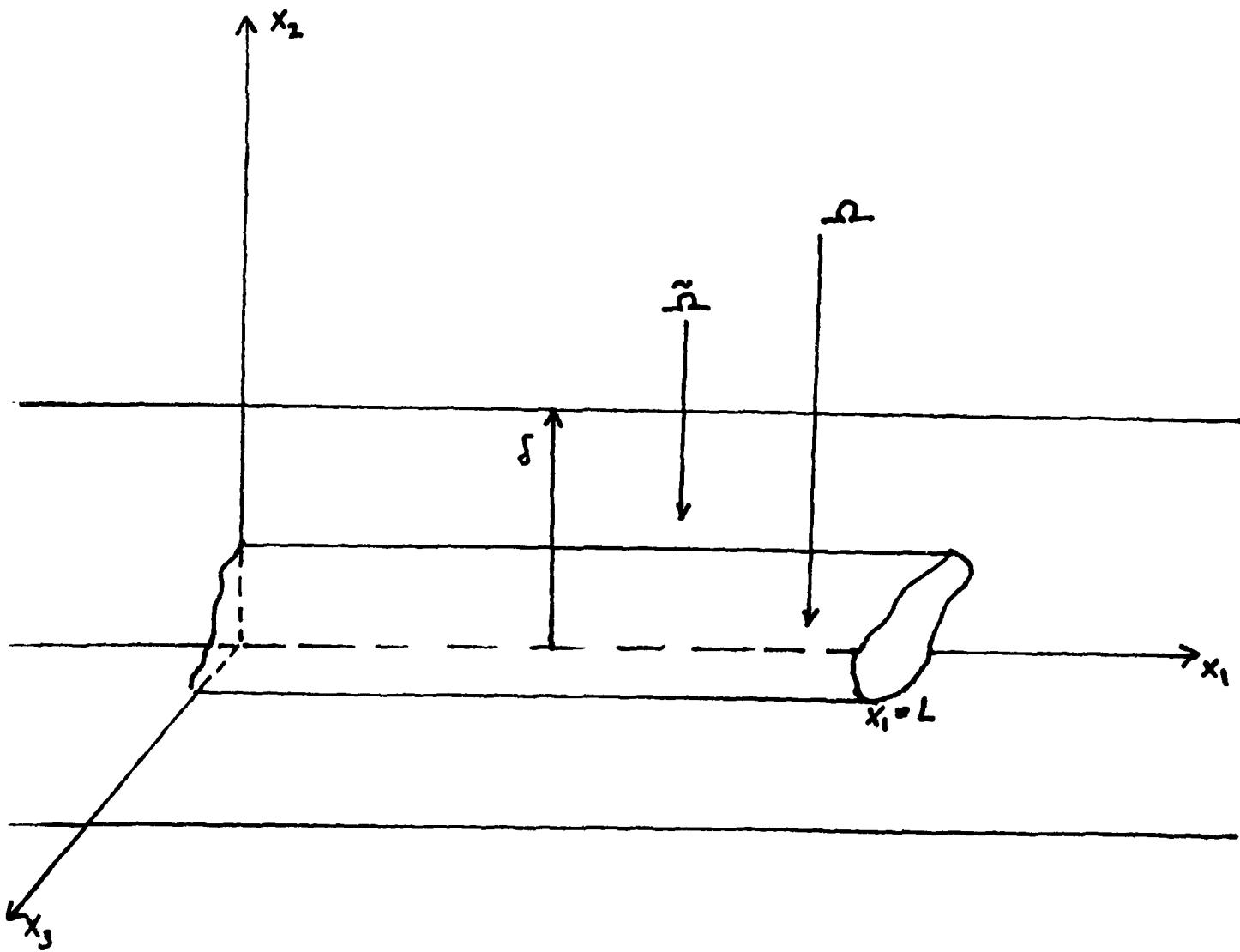


Figure 1

Specifically, we take for Ω the finite cylinder

$$(1.17) \quad \Omega = \{(x_1, x_2, x_3) \mid x_i \text{ real}, i = 1, 2, 3, 0 \leq x_1 \leq L,$$

$$\delta(x_2 x_3) = C_1(\text{const.})\}$$

with generators parallel to the x_1 axis and we assume that for some small $\epsilon > 0$

$$\Omega \cap \{(x_1, x_2, x_3) \mid x_1 = L', 0 \leq L' \leq L\}$$

$$\subseteq \{(x_1, x_2, x_3) \mid x_1 = L', 0 \leq L' \leq L, x_2^2 + x_3^2 \leq \epsilon^2\}.$$

For $\tilde{\Omega}$ we then take the (infinite) circular cylinder

$$(1.18) \quad \tilde{\Omega} = \{(x_1, x_2, x_3) \mid -\infty < x_1 < \infty, x_2^2 + x_3^2 = \delta^2, \delta > \epsilon > 0\}$$

and, in accordance with the boundary conditions (1.16a,b), we assume that the annular region $\tilde{\Omega}/\Omega$ between the dielectric rod and the circular cylinder is filled with a perfect conductor; in Ω the dielectric media is assumed to be governed by the constitutive hypothesis (1.5). Finally, we will assume that the entire configuration in Figure 1. is subjected to an applied electric field which is perpendicular to the $x_1 x_3$ plane and, hence, orthogonal to the axis of the dielectric; specifically, we assume that

$$(1.19) \quad \underline{E}(x, t) = (0, E_2(x_1, t), 0), \quad 0 \leq x_1 \leq L$$

with $E_2 \geq 0$ for $x_2 \in [0, L]$; of course, in $\tilde{\Omega}/\Omega$ we must have $\underline{E} = \underline{0}$. In order to proceed with the reduction of the evolution equations (1.7), which corresponds to the situation at hand, we will need some additional assumptions relative to the constitutive function λ ; specifically, the hypotheses on λ which will hold throughout the rest of this section are

$$(\lambda 1) \quad \lambda \in C^1(\mathbb{R}^3; [0, \infty)), \lambda(\xi) > 0, \forall \xi \neq 0$$

$$(\lambda 2) \quad |\lambda'(\xi)| < \infty, \forall \xi \in \mathbb{R}^3$$

$$(\lambda 3) \quad 0 < \zeta \hat{\lambda}'(\zeta) + \hat{\lambda}(\zeta) < \infty, \forall \zeta \in \mathbb{R}^3, \zeta \neq 0,$$

where $\hat{\lambda}(\zeta) \equiv \lambda((0, \zeta, 0)), \zeta \in \mathbb{R}^3$. By (λ1) and the definition of $\hat{\lambda}$ it is immediate that $\hat{\lambda} \in C^1(\mathbb{R}^3; [0, \infty))$.

We now proceed with the reduction of the nonlinear evolution equations (1.7). In view of (1.5₁), (1.19), in Ω

$$(0, E_2(0)) = \lambda(\underline{D})(D_1, D_2, D_3)$$

from which it follows that, in Ω , $D_1 = D_3 = 0$ and $E_2(x_1, t) = \lambda(\underline{D})D_2(x_1, x_2, x_3, t)$.

However, $\operatorname{div} \underline{D} = \frac{\partial D_2}{\partial x_2} = 0$ so that, for each $t \geq 0$, D_2 can depend, at most, on x_1, x_3 . As E_2 depends only on x_1

$$\begin{aligned} \frac{\partial E_2}{\partial x_3} &= \frac{\partial}{\partial x_3} (\lambda(\underline{D})D_2(x_1, x_3, t)) \\ &= \frac{\partial}{\partial x_3} (\lambda(0, D_2, 0)D_2(x_1, x_3, t)) \\ &= \frac{\partial}{\partial x_3} (\hat{\lambda}(D_2(x_1, x_3, t))D_2(x_1, x_3, t)) \\ &= \frac{\partial D_2}{\partial x_3} (\hat{\lambda}'(D_2)D_2 + \hat{\lambda}(D_2)) = 0. \end{aligned}$$

By hypothesis (λ3) it then follows that $\frac{\partial D_2}{\partial x_3} = 0$ and, thus, in Ω

$$(1.20) \quad \underline{D}(x, t) = (0, D_2(x_1, t), 0)$$

In view of (1.20), not only is $\operatorname{div} \underline{D} = 0$ automatically satisfied in Ω , but,

as is easily verified, so are the nonlinear evolution equations (1.7) for $i = 1, 3$, i.e.,

$$\frac{\partial}{\partial x_i} (\text{grad } \lambda(D) \cdot D) = \frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_2} \hat{\lambda}(D_2(x_1, t)) \cdot D_2(x_1, t) \right)$$

$$= 0, \quad i = 1, 2, 3$$

while $(D_i)_{ttt} = \nabla^2(\lambda(D)D_i) \equiv 0$ for $i = 1, 3$. For $i = 2$ we then obtain, for $0 \leq x_1 \leq L$, and $0 \leq t < T$,

$$(1.21) \quad \mu \frac{\partial^2 D_2}{\partial t^2}(x_1, t) = \nabla^2[\hat{\lambda}(D_2(x_1, t)) D_2(x_1, t)]$$

$$= \frac{\partial^2}{\partial x_1^2} [\hat{\lambda}(D_2(x_1, t)) D_2(x_1, t)].$$

We now turn our attention to the boundary conditions (1.16a,b); in order to simplify the exposition we will assume that $\sigma(x)$, the surface charge density at $x \in \partial\Omega$, vanishes on the planar faces of the cylindrical region Ω at $x_1 = 0$ and $x_1 = L$. Clearly, (Figure 1.), on the planar boundary at $x_1 = 0$, $y = (-1, 0, 0)$ and $t = (0, 1, 0)$. By (1.16a), therefore, with $\sigma(x) = 0$ for $x_1 = 0$, $\delta(x_1, x_3) = c_1$,

$$(1.22) \quad D(x, t) \cdot y \Big|_{\substack{x_1 = 0 \\ 0 \leq t < T}} = [(0, D_2(x_1, t), 0) \cdot (-1, 0, 0)] \Big|_{\substack{x_1 = 0 \\ 0 \leq t < T}} = 0$$

is trivially satisfied and an analogous result holds at $x_1 = L$ where $y = (1, 0, 0)$. In order to satisfy the boundary condition (1.16b) along the planar face at $x_1 = 0$, for $0 \leq t < T$, we require that

$$(1.23) \quad \lambda(D(\xi, t)) D(\xi, t) \cdot \xi \Big|_{\begin{array}{l} x_1 = 0 \\ 0 \leq t < T \end{array}} \\ = (0, \hat{\lambda}(D_2(x_1, t)) D_2(x_1, t), 0) \cdot (0, 1, 0) \\ = \hat{\lambda}(D_2(x_1, t)) D_2(x_1, t) = 0$$

In view of our assumption that $\lambda(\xi) > 0$, $\forall \xi \neq 0$, we have $\hat{\lambda}(\rho) > 0$ for $\rho \neq 0$. It then follows from (1.23) that the boundary condition (1.16b) will be satisfied along the planar surface at $x_1 = 0$, for $0 \leq t < T$, if $D_2(0, t) = 0$, $0 \leq t < T$; in an analogous fashion it follows that $D_2(L, t) = 0$, $0 \leq t < T$. In view of our assumptions relative to the nature of the medium in $\tilde{\Omega}/\Omega$ we also have $D_2 = 0$, $0 \leq t < T$, for $x_1 \in (-\infty, 0)$ and $x_1 \in (L, \infty)$. Thus $\text{supp } \{D_2\} = (0, L)$.

We now set $x_1 \equiv x$, $D_2 \equiv u$. Then, for the physical situation described above, the initial-boundary value problem associated with the coupled system of nonlinear evolution equations (1.7) reduces to the following nonlinear, one-dimensional, initial-boundary value problem on the x axis: find $u = u(x, t)$, $0 \leq x \leq L$, $0 \leq t < T$, such that

$$(1.24) \quad \left\{ \begin{array}{l} \mu \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2}{\partial x^2} (u \hat{\lambda}(u)), (x, t) \in [0, L] \times [0, T] \\ u(x, 0) = u_0(x), u_t(x, 0) = v_0(x), 0 \leq x \leq L \\ u(0, t) = 0, u(L, t) = 0, 0 \leq t < T \end{array} \right.$$

where $\mu > 0$, $\hat{\lambda}$ satisfies the hypotheses $(\lambda 1) - (\lambda 3)$, and for $0 \leq t < T$, $u(x, t) \equiv 0$ for $x < 0$, $x > L$.

Remarks

- (i) we note here an equivalent form for the one-dimensional non-linear

wave equation (1.24₁), i.e., as

$$\begin{aligned}\frac{\partial^2}{\partial x^2} (u \hat{\lambda}(u)) &= \frac{\partial}{\partial x} (u u_x \hat{\lambda}'(u) + u_x \hat{\lambda}(u)) \\ &= \frac{\partial}{\partial x} \left([u \hat{\lambda}'(u) + \hat{\lambda}(u)] \frac{\partial u}{\partial x} \right)\end{aligned}$$

(1.24₁) may be written with the spatial part of the equation in divergence form

$$\mu \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} (\Lambda(u) \frac{\partial u}{\partial x}), \quad (x,t) \in [0,L] \times [0,T]$$

where, in view of (λ3)

$$\Lambda(\zeta) = \zeta \hat{\lambda}'(\zeta) + \hat{\lambda}(\zeta) > 0, \quad \forall \zeta \in (-\infty, \infty), \quad \zeta \neq 0$$

(ii) The initial-boundary value problem (1.24) can be (formally) extended to a pure-initial value problem on $\mathbb{R}^1 \times [0,T]$ by extending the dielectric rod occupying the configuration Ω into the perfect conductor, i.e., we may think of having extended

$$\Omega \cup \tilde{\Omega} = \{(x_1, x_2, x_3) \mid -\infty < x_1 < \infty, f(x_2, x_3) = C_1\}.$$

Then $\tilde{\Omega} \subset \tilde{\Omega}$ and $u(x,t) \in D_2(x_1, t)$ satisfies (1.24₁) for $-\infty < x < \infty$; the extension of (1.24) to a pure initial-value problem on $\mathbb{R}^1 \times [0,T]$ is carried out in a rigorous fashion in §3, after (1.24) has first been transformed into an initial-boundary value problem for an equivalent quasilinear first order system, and involves extending $u_0(x)$, $u(x,t)$, as odd functions to $(-L,L)$, $v(x,t) \equiv \int_{-\infty}^x u_t(y,t) dy$ as an even function to $(-L,L)$, and then continuing these functions periodically to all of \mathbb{R}^1 with period $2L$.

2. Global Nonexistence of Electric Induction Fields

In this section we will demonstrate that under the additional hypothesis on the constitutive function $\hat{\lambda}(\zeta) = \lambda((0, \zeta, 0))$,

(λ4) For all $\zeta \in R^1$ and some $\alpha > 2$

$$\alpha \int_0^\zeta \rho \hat{\lambda}(\rho) d\rho \geq \zeta \hat{\lambda}(\zeta),$$

smooth global solutions of (1.24), i.e., solutions of (1.24) on $[0, L] \times [0, T]$, for all $T > 0$, will not, in general, exist; in fact, we will show that under relatively mild assumptions on the initial data, the $L^2(0, L)$ norm of $u(x, t)$ must be bounded from below by a real-valued nonnegative function of t which becomes infinite as $t \rightarrow t_\infty < \infty$. Some growth estimates for solutions of the initial-boundary value problem (1.24), which are valid on the maximal time-interval of existence, will also be derived. In §3, under stronger assumptions on $\hat{\lambda}(\zeta)$ than that represented by (λ4), we will demonstrate that smooth solutions of (1.24) cannot exist globally due to finite-time breakdown of the space-time gradient $(u_x(x, t), u_t(x, t))$.

Before proceeding with the analysis, let us note that if we set $\psi(\zeta) = \zeta \hat{\lambda}(\zeta)$, $\zeta \in R^1$, and $\Sigma(\zeta) = \int_0^\zeta \psi(\rho) d\rho$ then $\zeta \hat{\lambda}(\zeta) = \Sigma'(\zeta)$ and hypothesis (λ4) is equivalent to

(λ4) For all $\zeta \in R^1$ and some $\alpha > 2$ $\alpha \Sigma(\zeta) \geq \zeta \Sigma'(\zeta)$.

The proof of the global nonexistence property claimed above now proceeds via a series of lemmas, the first of which is just an energy conservation theorem for the solutions of (1.24), i.e.,

Lemma 2. If we define the total energy $E(t)$ of the system (1.24) by

$$(2.1) \quad E(t) = \frac{\mu}{2} \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right)^2 dx + \int_0^L \left(\int_0^{u(x, t)} \rho \lambda(\rho) d\rho \right) dx,$$

then for as long as smooth solutions of (1.24) exist,

$$(2.2) \quad E(t) = \frac{\mu}{2} \int_0^L \left(\int_{-\infty}^x v_0(y) dy \right)^2 dx + \int_0^L \left(\int_0^{u_0(x)} \rho \lambda(\rho) d\rho \right) dx$$

Proof. In view of the definitions of $\psi(\zeta)$, $\Sigma(\zeta)$,

$$(2.3) \quad E(t) = \frac{\mu}{2} \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right)^2 dx + \int_0^L \Sigma(u(x, t)) dx$$

Therefore,

$$\begin{aligned} (2.4) \quad \dot{E}(t) &= \mu \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right) \left(\int_{-\infty}^x u_{tt}(y, t) dy \right) dx \\ &\quad + \int_0^L \Sigma'(u(x, t)) u_t(x, t) dx \\ &= \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right) \left(\int_{-\infty}^x \frac{\partial^2}{\partial y^2} \psi(u(y, t)) dy \right) dx \\ &\quad + \int_0^L \Sigma'(u(x, t)) u_t(x, t) dx \\ &= \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right) \psi(u(x, t)),_x dx \\ &\quad + \int_0^L \Sigma'(u(x, t)) u_t(x, t) dx \end{aligned}$$

where we have used (1.24) and the fact that $u(x, t) \equiv 0$, $x < 0$, $t \geq 0$; i.e.,

$$\int_{-\infty}^x \frac{\partial^2}{\partial y^2} \psi(u(y, t)) dy = \psi(u(y, t)),_y |_{-\infty}^x$$

$$= \psi(u(x, t)),_x$$

$$- \lim_{\substack{\rho \rightarrow -\infty \\ \rho < 0}} (\psi(u(y, t)),_y |_{\rho})$$

$$= \psi(u(x, t)),_x$$

as $\psi(0) = 0$ by virtue of (λ1) and the definition of ψ . Therefore

$$(2.5) \quad \dot{E}(t) = \int_0^L \frac{\partial}{\partial x} [\psi(u(x,t)) \int_{-\infty}^x u_t(y,t)] dx \\ - \int_0^L \psi(u(x,t)) u_t(x,t) dx + \int_0^L \Sigma'(u(x,t)) u_t(x,t) dx \\ = \psi(u(L,t)) \int_{-\infty}^L u_t(y,t) dy - \psi(u(0,t)) \int_{-\infty}^0 u_t(y,t) dy$$

as $\psi(\zeta) = \Sigma'(\zeta)$, $\forall \zeta \in R^1$, by definition. As $\psi(0) = 0$, the boundary conditions (1.24)₃ now imply that $\dot{E}(t) = 0$, $0 \leq t < T$, and (2.2) then follows by integration over $[0,t]$, the definition of $E(t)$, and the initial conditions (1.24)₂.

Q.E.D.

Our next lemma is concerned with establishing a certain differential inequality for a real-valued nonnegative functional defined on solutions $u(x,t)$ of the initial-boundary value problem (1.24); namely, we have

Lemma 3. Let $u(x,t)$, $(x,t) \in [0,L] \times [0,T]$ be a smooth solution of (1.24) and define

$$(2.6) \quad F(t) = \mu \int_0^L \left(\int_{-\infty}^x u(y,t) dy \right)^2 dx + \beta(t+t_0)^2$$

where $\beta, t_0 \geq 0$. If $\lambda^*(\zeta)$ satisfies (λ1) - (λ4), then for $0 \leq t < T$

$$(2.7) \quad FF'' - (\gamma+1)F'^2 \geq -2(2\gamma+1)F(\beta + 2E(0))$$

where $\gamma = \frac{\alpha-2}{4} > 0$ (with α the constant which arises in the constitutive assumption (λ4)) and $E(0)$, the initial energy, is given by the right-hand side of (2.2).

Proof. By direct differentiation we have

$$(2.8) \quad F'(t) = 2\mu \int_0^L \left(\int_{-\infty}^x u(y, t) dy \right) \left(\int_{-\infty}^x u_t(y, t) dy \right) dx + 2\beta(t+t_0)$$

and

$$(2.9) \quad F''(t) = 2\mu \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right)^2 dx \\ + 2\mu \int_0^L \left(\int_{-\infty}^x u(y, t) dy \right) \left(\int_{-\infty}^x u_{tt}(y, t) dy \right) dx + 2\beta.$$

Again, in view of (1.24₁), the definition of $\psi(\zeta)$, $\zeta \in R^1$, and the fact that $u(x, t) \equiv 0$, $x < 0$, $t \geq 0$, we have

$$(2.10) \quad F''(t) = 2\mu \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right)^2 dx \\ + 2 \int_0^L \left(\int_{-\infty}^x u(y, t) dy \right) \psi(u(x, t))_{,x} dx + 2\beta \\ = 2\mu \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right)^2 dx \\ + 2 \int_0^L \frac{\partial}{\partial x} \{ (\int_{-\infty}^x u(y, t) dy) \psi(u(x, t)) \} dx \\ - 2 \int_0^L u(x, t) \psi(u(x, t)) dx + 2\beta \\ = 2\mu \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right)^2 dx \\ - 2 \int_0^L u(x, t) \Sigma'(u(x, t)) dx + 2\beta$$

where we have again used the fact that $\psi(u(0, t)) = \psi(u(L, t)) = 0$, $0 \leq t < T$.

By adding and subtracting $2\alpha \int_0^L \Sigma(u(x, t)) dx$ on the right-hand side of the last line in (2.10) we obtain

$$(2.11) \quad F''(t) = 2\mu \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right)^2 dx - 2\alpha \int_0^L \Sigma(u(x, t)) dx \\ + 2 \int_0^L (\alpha \Sigma(u(x, t)) - u(x, t) \Sigma'(u(x, t))) dx + 2\beta \\ \geq 2\mu \int_0^L \left(\int_{-\infty}^x u_t(y, t) dy \right)^2 dx - 2\alpha \int_0^L \Sigma(u(x, t))$$

where we have used the hypothesis (λ4) in the form given by (λ4). However, in

view of the definitions of $E(t)$, i.e. (2.1), and $\Sigma(\zeta)$, $\zeta \in \mathbb{R}^1$, the inequality in (2.11) may be replaced by

$$\begin{aligned}
 (2.12) \quad F''(t) &\geq 2\mu \int_0^L \left(\int_{-\infty}^x u_t(y,t) dy \right)^2 dx \\
 &\quad - 2\alpha [E(t) - \frac{\mu}{2} \int_0^L \left(\int_{-\infty}^x u_t(y,t) dy \right)^2 dx] + 2\beta \\
 &= (2+\alpha)\mu \int_0^L \left(\int_{-\infty}^x u_t(y,t) dy \right)^2 dx \\
 &\quad - 2\alpha E(0) + 2\beta
 \end{aligned}$$

where we have used the energy conservation result of Lemma 2. Finally, we rewrite the last inequality in (2.12) in the form

$$\begin{aligned}
 (2.13) \quad F''(t) &\geq (2+\alpha) [\mu \int_0^L \left(\int_{-\infty}^x u_t(y,t) dy \right)^2 dx + \beta] \\
 &\quad - \alpha [\beta + 2E(0)]
 \end{aligned}$$

Combining (2.8), (2.13) and (2.6) we now obtain

$$\begin{aligned}
 (2.14) \quad FF'' &- \left(\frac{\alpha+2}{4} \right) F'^2 \\
 &\geq (2+\alpha) \left[\mu \int_0^L \left(\int_{-\infty}^x u(y,t) dy \right)^2 dx + \beta(t + t_0)^2 \right] \\
 &\quad \times \left[\mu \int_0^L \left(\int_{-\infty}^x u_t(y,t) dy \right)^2 dx + \beta \right] \\
 &\quad - \alpha F(\beta + 2E(0)) \\
 &\quad - (2+\alpha) \left[\mu \int_0^L \left(\int_{-\infty}^x u(y,t) dy \right) \left(\int_{-\infty}^x u_t(y,t) dy \right) dx \right. \\
 &\quad \left. + \beta(t + t_0) \right]^2
 \end{aligned}$$

$$\begin{aligned}
&= (2+\alpha) \left\{ \left([\mu \int_0^L (\int_{-\infty}^x u(y,t) dy)^2 dx + \beta(t+t_0)^2] \right. \right. \\
&\quad \times [\mu \int_0^L (\int_{-\infty}^x u_t(y,t) dy)^2 dx + \beta] \Big) \\
&\quad - \left(\mu \int_0^L (\int_{-\infty}^x u(y,t) dy) (\int_{-\infty}^x u_t(y,t) dy) dx \right. \\
&\quad \left. \left. + \beta(t+t_0)^2 \right) \right\} \\
&\quad - \alpha F(\beta + 2E(0)).
\end{aligned}$$

By virtue of the Cauchy-Schwarz inequality the { } expression in the last inequality in (2.14) is nonnegative for all $t, 0 \leq t < T$, and, therefore,

$$(2.15) \quad FF'' - \left(\frac{\alpha+2}{\mu} \right) F'^2 \geq -\alpha F(\beta + 2E(0)), \quad 0 \leq t < T$$

The required result, i.e., (2.7) now follows directly from (2.15) if we set $\gamma = (\alpha-2)/4$.

Q.E.D.

Global nonexistence of solutions to the initial-boundary value problem (1.24) can now easily be shown to be a consequence of the differential inequality (2.7) under various assumptions on the initial energy $E(0)$ and the initial data $u_0(x), v_0(x)$. To simplify the discussion we introduce the notation

$$(2.16a) \quad I(u_0) = \mu \int_0^L (\int_{-\infty}^x u_0(y) dy)^2 dx$$

$$(2.16b) \quad J(u_0, v_0) = 2\mu \int_0^L (\int_{-\infty}^x u_0(y) dy) (\int_{-\infty}^x v_0(y) dy) dx$$

Our first result then assumes the following form

Theorem 1. Let $u(x,t)$ be a solution of (1.24) and assume that the constitutive function $\lambda^*(\zeta) = \lambda((0,\zeta,0))$ satisfies $(\lambda 1) - (\lambda 4)$. If $J(u_0, v_0) > 0$ and

$$(2.17) \quad \int_0^L \left(\int_0^{u_0(x)} \psi(\rho) d\rho \right) dx \leq - \frac{\mu}{2} \int_0^L \left(\int_{-\infty}^x v_0(y) dy \right)^2 dx$$

then $\exists \kappa = \kappa(\mu; L) > 0$ and $t_\infty < \infty$ such that

$$(2.18) \quad \|u(t)\|_{L^2(0,L)}^2 \geq \kappa G(t), \quad 0 \leq t \leq t_{\max}$$

where $[0, t_{\max})$ denotes the maximal interval of existence of $u(x,t)$ and

$$\lim_{t \rightarrow t_\infty} G(t) = +\infty.$$

Remark: As a consequence of (2.18), and the fact that $G(t)$ tends to $+\infty$ as $t \rightarrow t_\infty$, it follows that $t_{\max} \leq t_\infty$, i.e., that the maximal interval of existence of $u(x,t)$ is finite; it may happen, however, that $t_{\max} < t_\infty$ and thus, without stronger assumptions on $\lambda^*(\zeta)$, we cannot conclude that $u(x,t)$, or its derivatives, blow-up in an appropriate norm as $t \rightarrow t_\infty$ (See J. Ball [10], [11] for a relevant discussion of the relationship between global nonexistence and finite-time blow-up theorems for solutions of nonlinear evolution equations).

Proof (Theorem 1). In view of (2.17), $E(0) \leq 0$. Thus, if we set $\beta = 0$ in (2.7) this inequality reduces to

$$(2.19) \quad F_0(t) F_0''(t) - (\gamma+1) F_0'^2(t) \geq 0, \quad 0 \leq t \leq t_{\max},$$

$F_0(t) = \mu \int_0^L \left(\int_{-\infty}^x u(y,t) dy \right)^2 dx$. But (2.19) is easily seen to be equivalent to

$$(2.20) \quad (F_0^{-\gamma})''(t) \leq 0, \quad 0 \leq t \leq t_{\max}$$

Two successive integrations of (2.20) yield

$$(2.21) \quad F_0^{-\gamma}(t) \leq -\gamma F_0^{-\gamma-1}(0) F_0'(0)t + F_0^{-\gamma}(0), \quad 0 \leq t \leq t_{\max}$$

or, as $\gamma > 0$, $F_0(t) > 0$

$$(2.22) \quad F_0(t) \geq \left[\frac{F_0^\gamma(0)}{1 - \gamma \left(\frac{F_0'(0)}{F_0(0)} \right) t} \right]^{\frac{1}{\gamma}} \equiv G(t), \quad 0 \leq t \leq t_{\max}$$

Clearly, $\lim_{t \rightarrow t_{\max}} G(t) = +\infty$ where

$$(2.23) \quad t_{\infty} = \frac{1}{\gamma} \left(\frac{F_0(0)}{F_0'(0)} \right) = \frac{1}{\gamma} \frac{I(u_0)}{J(u_0, v_0)} < \infty$$

Also, as $u(x, t) \equiv 0$, $x < 0$

$$\begin{aligned} \int_0^L (\int_{-\infty}^x u(y, t) dy)^2 dx &= \int_0^L (\int_0^x u(y, t) dy)^2 dx \\ &\leq \int_0^L x (\int_0^x u^2(y, t) dy) dx \\ &\leq (\int_0^L x^2 dx)^{\frac{1}{2}} (\int_0^L (\int_0^x u^2(y, t) dy)^2 dx)^{\frac{1}{2}} \\ &\leq \sqrt{\frac{L^3}{3}} \left(\int_0^L (\int_0^L u^2(y, t) dy)^2 dx \right)^{\frac{1}{2}} \\ &\leq \sqrt{\frac{L^3}{3}} \cdot \sqrt{L} \int_0^L u^2(y, t) dy \end{aligned}$$

and, therefore,

$$(2.24) \quad \|u(t)\|_{L^2(0, L)}^2 \geq \frac{\sqrt{3}}{L^2} \int_0^L (\int_{-\infty}^x u(y, t) dy)^2 dx$$

The growth estimate (2.18), valid for $0 \leq t \leq t_{\max}$, now follows directly from (2.22), (2.24), and the definition of $F_0(t)$, with $\kappa = \sqrt{3}/\mu L^2$.

Q.E.D.

There are several other situations in which the same basic conclusion, as that expressed by Theorem I, follows; we will examine two such sets of circumstances below which correspond to situations in which we have, respectively, $J(u_0, v_0) = 0$ and $J(u_0, v_0) < 0$, with $E(0) < 0$ in both cases. Suppose, first of all, that $E(0) < 0$ with $v_0(x) \geq 0$, $0 \leq x \leq L$; in this case we may choose $\beta = \beta_0$ such that $2E(0) + \beta_0^2 = 0$ and therefore (2.7) reduces to (2.19) with $F_0(t)$ replaced by

$$F(t; \beta_0, t_0) \equiv \mu \int_0^L (\int_{-\infty}^x u(y, t) dy)^2 dx + \beta_0(t+t_0)^2$$

Therefore, $F(t; \beta_0, t_0)$ satisfies, for $0 \leq t \leq t_{\max}$,

$$(2.25) \quad F(t; \beta_0, t_0) \geq \left[\frac{F^\gamma(0; \beta_0, t_0)}{1-\gamma \left(\frac{F'(0; \beta_0, t_0)}{F(0; \beta_0, t_0)} \right) t} \right]^{\frac{1}{\gamma}} \equiv H(t),$$

so that $\lim_{t \rightarrow t_\infty(t_0)} H(t) = +\infty$ where

$$(2.26) \quad t_\infty(t_0) = \frac{1}{\gamma} \frac{F(0; \beta_0, t_0)}{F'(0; \beta_0, t_0)}$$

$$= \frac{1}{\gamma} \left(\frac{I(u_0) + \beta_0^2 t_0^2}{2\beta_0 t_0} \right)$$

We note that, in view of our hypothesis,

$$(2.2) \quad \beta_0 = -2 \int_0^L (\int_0^{u_0(x)} \psi(\rho) d\rho) dx > 0$$

It is not difficult to show that the minimum value of $t_\infty(t_0)$ is achieved at

$$t_0 = \tilde{t}_0 \equiv \sqrt{\frac{T(u_0)}{\beta_0}}$$

and that

$$(2.28) \quad t_{\infty}(\tilde{t}_0) = \sqrt{\frac{I(u_0)}{\beta_0}} = \tilde{t}_0$$

Choosing $t_0 = \tilde{t}_0$ in (2.25) we have, therefore,

$$(2.29) \quad \mu \int_0^L \left(\int_{-\infty}^x u(y, t) dy \right)^2 dx + \beta_0(t + \tilde{t}_0)^2$$

$$\geq \left[\frac{(I(u_0) + \beta_0 \tilde{t}_0^2)^{\gamma}}{1 - \frac{1}{t_{\infty}(\tilde{t}_0)} t} \right]^{\frac{1}{\gamma}}$$

$$\geq \frac{I(u_0)}{\left(1 - \sqrt{\frac{\beta_0}{I(u_0)}} t \right)^{\frac{1}{\gamma}}}$$

for $0 \leq t \leq t_{\max}$. In view of (2.24), (2.28) we then have the growth estimate

$$(2.30) \quad \frac{1}{\kappa} \|u(t)\|_{L^2(0, L)}^2 + \beta_0 \left(t + \sqrt{\frac{I(u_0)}{\beta_0}} \right)^2$$

$$\geq \frac{I(u_0)}{\left(1 - \sqrt{\frac{\beta_0}{I(u_0)}} t \right)^{\frac{1}{\gamma}}},$$

for $0 \leq t \leq t_{\max} \leq \sqrt{\frac{I(u_0)}{\beta_0}}$, where β_0 is given by (2.27) and $I(u_0)$ by (2.16a).

The estimate (2.30) establishes global nonexistence of solutions to the initial-boundary value problem, under the hypotheses (λ1) - (λ4), for the case where the initial data satisfy $v_0(x) \equiv 0$, $0 \leq x \leq L$, and $\int_0^L \left(\int_0^{u_0(x)} \rho \lambda^*(\rho) d\rho \right) dx < 0$.

Having examined the cases where $E(0) \leq 0$ with $J(u_0, v_0) > 0$ and $E(0) < 0$ with $J(u_0, v_0) = 0$, $v_0(x) \geq 0$, $0 \leq x \leq L$, we now want to look at the situation where $E(0) < 0$, i.e.,

$$(2.31) \quad \int_0^L \left(\int_0^{u_0(x)} \psi(\rho) d\rho \right) dx < - \frac{\mu}{2} \int_0^L \left(\int_{-\infty}^x v_0(y) dy \right)^2 dx$$

and $J(u_0, v_0) < 0$. In this case we may again choose $\beta = \bar{\beta}_0$ such that $2E(0) + \bar{\beta}_0 = 0$, so that $F(t; \bar{\beta}_0, t_0)$ satisfies (2.25), with $\beta_0 \rightarrow \bar{\beta}_0$, for $0 \leq t \leq t_{\max}$. We note that we now have

$$(2.32) \quad t_{\infty}(t_0) = \frac{I(u_0) + \bar{\beta}_0 t_0^2}{2\bar{\beta}_0 t_0 - |J(u_0, v_0)|}$$

where

$$(2.33) \quad \bar{\beta}_0 = -\mu \int_0^L \left(\int_{-\infty}^x v_0(y) dy \right)^2 dx - 2 \int_0^L \left(\int_0^{u_0(x)} \rho \lambda^*(\rho) d\rho \right) dx > 0$$

and thus we must choose $t_0 \geq \bar{t}_0$ where

$$(2.34) \quad \bar{t}_0 > \frac{1}{2\bar{\beta}_0} |J(u_0, v_0)|$$

It is a relatively simple matter to show that $t_{\infty}(t_0)$ achieves a minimum at

$$(2.35) \quad t_0 = \bar{t}_0 \equiv \frac{1}{2\bar{\beta}_0} \left(|J(u_0, v_0)| + \sqrt{|J(u_0, v_0)|^2 + 4\bar{\beta}_0 I(u_0)} \right)$$

If we denote $t_{\infty}(\bar{t}_0) = \bar{t}_{\infty}$ then we have the estimate

$$\begin{aligned} (2.36) \quad & \mu \int_0^L \left(\int_{-\infty}^x u(y, t) dy \right)^2 dx + \bar{\beta}_0 (t + \bar{t}_0)^2 \\ & \geq \left[\frac{(I(u_0) + \bar{\beta}_0 \bar{t}_0^2)^{\gamma}}{1 - \bar{t}_{\infty}^{-1} t} \right]^{\frac{1}{\gamma}} \\ & \geq \frac{I(u_0)}{\left(1 - \bar{t}_{\infty}^{-1} t \right)^{\frac{1}{\gamma}}} \end{aligned}$$

for $0 \leq t \leq t_{\max}$ and the companion estimate

$$2.37) \quad \frac{1}{\kappa} \|u(t)\|_{L^2(0,L)}^2 + \beta_0(t+\bar{t}_0)^2$$

$$\geq \frac{I(u_0)}{(1 - \bar{t}_{\infty}^{-1} t)^{\frac{1}{\gamma}}}$$

for $0 \leq t \leq t_{\max} \leq \bar{t}_{\infty}$ and global nonexistence of solutions to the initial-boundary value problem (1.24) follows as in the previous cases. We may summarize the two results corresponding to the situation where $E(0) < 0$ as

Theorem II. Let $u(x,t)$ be a solution of (2.24) and assume that the constitutive function $\lambda^*(\rho) = \lambda((0,\rho,0))$ satisfies (λ1) - (λ4). Then

(i) If $v_0(x) \equiv 0$, $0 \leq x \leq L$, and $\int_0^L (\int_0^{u_0(x)} \rho \lambda^*(\rho) d\rho) dx < 0$,
then $u(x,t)$ satisfies, for $0 \leq t \leq t_{\max} \leq \sqrt{\frac{I(u_0)}{\beta_0}}$, the growth estimate (2.30)
where β_0 is given by (2.27).

(ii) If the initial data $(u_0(x), v_0(x))$ satisfy (2.31) and

$$\int_0^L (\int_{-\infty}^x u_0(y) dy) (\int_{-\infty}^x v_0(y) dy) dx < 0,$$

then $u(x,t)$ satisfies, for $0 \leq t \leq t_{\max} \leq \bar{t}_{\infty}$, the growth estimate (2.37),
where β_0 is given by (2.33), \bar{t}_0 by (2.35), (2.16b), and $\bar{t}_{\infty} = t_{\infty}(\bar{t}_0)$ where
 $t_{\infty}(t_0)$ is given by (2.32). In both cases (i) and (ii) above the respective
estimates (2.30), (2.37) imply that solutions of (1.24) cannot exist globally,
i.e., for $t \in [0, \infty)$.

Remarks. Results analogous to those established in Theorems I and II for the situations where $E(0) \leq 0$ with $u_0^2 + v_0^2 \neq 0$ and $E(0) < 0$, respectively,

can also be established for the case where $E(0) = 0$, with $u_0 = v_0 = 0$, and for certain situations in which $E(0) > 0$; we will not pursue a further discussion of these situations here but, rather, refer the reader to the recent work of Knops, Levine, and Payne [12] on growth estimates and global nonexistence theorems in nonlinear elastodynamics where there also arise differential inequalities of the same formal form as (2.7) for which associated growth estimates are derived in cases where the term corresponding to our $E(0)$, in those estimates, is either positive or zero.

3. Riemann Invariants and Finite-Time Breakdown of the Electric Induction Field.

In this section we offer a brief demonstration of the fact that under a slightly different set of assumptions on $\hat{\lambda}(\zeta)$, $\zeta \in R^1$, than those represented by $(\lambda_1) - (\lambda_{11})$, it is possible to apply the Riemann Invariant argument of Lax [13] so as to conclude that finite-time breakdown of $u_t + \frac{1}{\sqrt{\mu}} \nabla \psi(u) u_x$ must occur, where $u(x,t)$ is a solution of the initial-boundary value problem (1.24).

In [13] Lax considers the nonlinear initial-boundary value problem on $[0,L] \times [0,\infty)$

$$(3.1) \quad \begin{cases} y_{tt}(x,t) = K^2(y_x) y_{xx}(x,t), \\ y(x,0) = y_0(x), \quad y_t(x,0) = 0; \quad 0 \leq x \leq L \\ y(0,t) = y(L,t) = 0, \quad t > 0 \end{cases}$$

This problem may be extended to a pure-initial value problem on $R^1 \times [0, \infty)$ by extending $y_0(\cdot)$, $y(\cdot, t)$ as odd functions to $(-L, L)$ and then periodically, to all of R^1 , with period $2L$. By setting $U = y_x$, $V = y_t$ the resulting extended initial-value problem on R^1 is then easily seen to be equivalent to a pure initial-value problem for a coupled quasilinear system on $R^1 \times [0, \infty)$, i.e.,

$$(3.2) \quad \begin{pmatrix} U \\ V \end{pmatrix}_{,t} + \begin{pmatrix} 0 & -1 \\ -K^2(U) & 0 \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix}_{,x} = 0$$

$$\begin{pmatrix} U(x,0) \\ V(x,0) \end{pmatrix} = \begin{pmatrix} \tilde{y}'_0(x) \\ 0 \end{pmatrix} \quad \left\{ \begin{array}{l} \text{\tilde{y}_0 the extension} \\ \text{of y_0 to R^1} \end{array} \right\}$$

The eigenvalues and eigenvectors associated with the system (3.2) are, respectively,

$$(3.3) \quad \begin{Bmatrix} \delta(U) \\ \rho(U) \end{Bmatrix} = \pm K(U) \quad \text{and} \quad \begin{bmatrix} 1 \\ \pm K(U) \end{bmatrix}$$

and thus the system is hyperbolic if and only if $K^2(\xi) > 0, \forall \xi \in R^1$. Also, the system may be diagonalized in a familiar way so as to yield the system

$$(3.4) \quad \begin{aligned} V' + K(U)U' &= 0 \\ V'' - K(U)U'' &= 0 \end{aligned}$$

where $' \equiv \frac{\partial}{\partial t} - K(U) \frac{\partial}{\partial x}$ and $'' \equiv \frac{\partial}{\partial t} + K(U) \frac{\partial}{\partial x}$ denote, respectively, differentiation along the right and left-hand characteristics defined by the ordinary differential equations $\frac{dx}{dt} = \pm K(U)$. Using (3.4) one then shows in the standard way that the Riemann Invariants

$$(3.5) \quad \begin{aligned} R(U,V) &= V + \int_0^U K(\zeta) d\zeta \\ S(U,V) &= V - \int_0^U K(\zeta) d\zeta \end{aligned}$$

satisfy $R' = S'' = 0$, i.e., that they are constant along the respective characteristic curves. It is shown in [13] that with a suitable choice of $H = H(R, A)$, the function $Z = \exp(A)R_x$ satisfies $Z' = -[(\exp(-H))\delta_R]Z^2$ where $\delta_R = \frac{\partial K}{\partial U}/2K(U)$ so that Z , and hence R_x , must breakdown (blow-up) in finite time if $\exists C > 0$ such that $|(\exp(-H))\delta_R| \geq C$; this last condition, on the other hand, turns out to be a consequence of the assumption that $\exists \bar{\epsilon} > 0$ such that $|\partial K/\partial U| \geq \bar{\epsilon} > 0, \forall U \in R^1$. Finite-time breakdown for R_x then implies finite-time breakdown for at least one of the second-order derivatives y_{xx}, y_{tt} of the solution $y(x, t)$ to the nonlinear initial-boundary value problem (3.1), as

$$\begin{aligned} R_x &= R_U U_x + R_V V_x \\ &= K(U)U_x + V_x \\ &= K(y_x)y_{xx} + y_{xt} \end{aligned}$$

Suppose that we now reconsider the initial-boundary value problem (1.24) and recall that as a consequence of the fact that $u(x,t) \equiv 0$, $(x,t) \in (-\infty, 0] \times [0, \infty)$

$$\mu \int_{-\infty}^x u_{tt}(y,t) dy = \psi(u(x,t)),_x = \psi'(u) u_x(x,t).$$

If we set

$$(3.6) \quad v(x,t) = \int_{-\infty}^x u_t(y,t) dy, \quad t \geq 0$$

then, clearly, $v_x(x,t) = u_t(x,t)$ and $v_t(x,t) = \int_{-\infty}^x u_{tt}(y,t) dy = \frac{1}{\mu} \psi'(u) u_x(x,t)$.

Also, $u(x,0) = u_0(x)$, $v(x,0) = \int_{-\infty}^x u_t(y,0) dy = \int_{-\infty}^x v_0(y) dy$, $0 \leq x \leq L$. Therefore, the initial-boundary value problem (1.24) for $u(x,t)$ is easily seen to be equivalent to the following initial-boundary value problem for the pair $(u(x,t), v(x,t))$:

$$(3.7) \quad \begin{cases} u_t - v_x = 0 \\ v_t - \frac{1}{\mu} \psi'(u) u_x = 0 \end{cases} \quad \begin{cases} 0 \leq x \leq L \\ t \geq 0 \end{cases}$$

$$\begin{cases} u(x,0) = u_0(x), \quad v(x,0) = \int_{-\infty}^x v_0(y) dy, \quad 0 \leq x \leq L \\ u(0,t) = u(L,t) = 0, \quad t \geq 0 \end{cases}$$

The system (3.7) is clearly of the same form as that considered by Lax [13], i.e., (3.2), if we assume that $v_0(x) \equiv 0$, $0 \leq x \leq L$; also, the initial-boundary value problem (3.7) may, in view of the above definition of $v(x,t)$, be extended to a pure-initial value problem on \mathbb{R}^1 if we

- (i) extend $u_0(\cdot)$ as an odd function to $(-L, L)$ and then periodically with period $2L$
- (ii) define, for $-L \leq x \leq 0$, $u(x,t) = -u(-x,t)$, $v(x,t) = v(-x,t)$, and then extend u and v to all of \mathbb{R}^1 as periodic functions with period $2L$.

The system (3.7) now assumes the following form

$$(3.8) \quad \begin{cases} \left(\begin{pmatrix} u \\ v \end{pmatrix}, t + \begin{pmatrix} 0 & -1 \\ -\frac{1}{\mu} \psi'(u) & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}, x \right) = 0 & \begin{cases} -\infty < x < \infty \\ t \geq 0 \end{cases} \\ \begin{pmatrix} u(x, 0) \\ v(x, 0) \end{pmatrix} = \begin{pmatrix} \tilde{u}_0(x) \\ 0 \end{pmatrix}, -\infty < x < \infty \end{cases}$$

when $\tilde{u}_0(x)$, $-\infty < x < \infty$, is the extension of $u_0(x)$ to \mathbb{R}^1 . In comparing (3.8) with (3.2) we clearly have the correspondence $K^2(\zeta) = \frac{1}{\mu} \psi'(\zeta)$, $\zeta \in \mathbb{R}^1$, and thus (3.8) is a hyperbolic system if and only if

$$\psi'(\zeta) = \zeta \hat{\lambda}'(\zeta) + \hat{\lambda}(\zeta) > 0, \quad \forall \zeta \in \mathbb{R}^1$$

which is precisely hypothesis ($\lambda 3$). The Riemann Invariants associated with the system (3.8) are, clearly, given by the expressions

$$(3.9) \quad \begin{cases} r(u, v) = v + \frac{1}{\sqrt{\mu}} \int_0^u \sqrt{\psi'(\rho)} d\rho \\ s(u, v) = v - \frac{1}{\sqrt{\mu}} \int_0^u \sqrt{\psi'(\rho)} d\rho \end{cases}$$

and they satisfy $r' = s' = 0$ along the respective characteristics given by

$$\frac{dx}{dt} = \pm \sqrt{\frac{\psi'(u)}{\mu}} \quad \text{where} \quad \dot{\tau} \equiv \frac{\partial}{\partial t} - \sqrt{\frac{\psi'}{\mu}} \frac{\partial}{\partial x} \quad \text{and} \quad \dot{\gamma} \equiv \frac{\partial}{\partial t} + \sqrt{\frac{\psi'}{\mu}} \frac{\partial}{\partial x}.$$

By the results in [13], which we have described above, finite-time breakdown (blow-up) of

$$(3.10) \quad \begin{aligned} r_x &= v_x + \frac{1}{\sqrt{\mu}} \sqrt{\psi'(u)} u_x \\ &= u_t + \frac{1}{\sqrt{\mu}} \sqrt{\psi'(u)} u_x \end{aligned}$$

will occur if, $\forall \zeta \in R^1$,

$$\begin{aligned} \left| \frac{d}{d\zeta} \left(\frac{1}{\sqrt{\mu}} \cdot \sqrt{\psi'(\zeta)} \right) \right| &= \frac{1}{2} \left| \frac{\psi''(\zeta)}{\sqrt{\mu\psi'(\zeta)}} \right| \\ &= \frac{1}{2\sqrt{\mu\psi'(\zeta)}} \cdot |\psi''(\zeta)| \geq \epsilon^* \end{aligned}$$

for some $\epsilon^* > 0$. Using the relationship between $\hat{\lambda}(\zeta)$ and $\psi(\zeta)$ this last condition is equivalent to the requirement that $\hat{\lambda}(\zeta), \forall \zeta \in R^1$, satisfy, for some $\epsilon > 0$,

$$(λ5) \quad |\zeta \hat{\lambda}''(\zeta) + 2\hat{\lambda}'(\zeta)| \geq \epsilon \sqrt{\zeta \hat{\lambda}'(\zeta) + \hat{\lambda}(\zeta)},$$

It also follows from the work of Lax [13] that

$$t_{\max} \approx \frac{4\sqrt{\mu}}{\max u_0'(x)} \cdot \frac{\sqrt{\psi'(0)}}{\psi''(0)}$$

As $\psi'(\zeta) = \zeta \hat{\lambda}'(\zeta) + \hat{\lambda}(\zeta)$ and $\psi''(\zeta) = \zeta \hat{\lambda}''(\zeta) + 2\hat{\lambda}'(\zeta)$, $\forall \zeta \in R^1$, we clearly must require that $\hat{\lambda}(\zeta)$ also satisfy the conditions

$$(λ6) \quad 0 < \hat{\lambda}(0) < \infty, \quad 0 < \hat{\lambda}'(0) < \infty, \quad |\hat{\lambda}''(0)| < \infty,$$

$$\text{in which case } t_{\max} \approx \frac{2\sqrt{\mu}}{\max u_0'(x)} \left(\frac{\hat{\lambda}(0)}{\hat{\lambda}'(0)} \right)$$

We summarize the above discussion as

Theorem III. Let $u(x,t)$ be a solution of (1.24) and assume that the constitutive function $\hat{\lambda}(\zeta) = \lambda((0,\zeta,0))$ is of class $C^2(R^1)$ and satisfies (λ3), (λ5) for some $\epsilon^* > 0$, and (λ6). Then, if $v_0(x) \equiv 0$, $0 \leq x \leq L$, the space-time gradient (u_x, u_t) must break-down in finite-time $t_{\max} \approx 2\mu^{1/2} \hat{\lambda}(0) / \max u_0'(x) \cdot \hat{\lambda}'(0)$.

Remarks. Different finite-time breakdown results for solutions of the initial-boundary value problem (1.24) may be gleaned from the work of MacCamy and Mizel [14]; the hypotheses of [14] do not, however, seem as well suited to the discussion of (1.24) as do those of Lax [13]. Finite-time breakdown results for initial-boundary value problems associated with the coupled system of equations (1.12) in $\mathbb{R}^3 \times [0, \infty)$ cannot, of course, be obtained via the use of Riemann Invariant type arguments as the applicability of such arguments is essentially restricted to one-dimensional situations; we hope to discuss the general problems of local and global existence, and global nonexistence of solutions for the coupled three-dimensional system (1.12) in future work. For recent work on the breakdown of solutions to nonlinear wave equations in spatial dimension $n > 1$ we refer the reader to John [15] and Payne and Sattinger [16]; a good source for work on the problem of proving global existence of solutions for nonlinear wave equations in dimension $n > 1$ is the recent work of Klainerman [17].

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20. Abstract cont.

initial-boundary value problem for the electric induction field cannot exist globally in time in the L^{∞} sense; under slightly stronger assumptions on the constitutive function), a standard Riemann Invariant argument may be applied to show that the space-time gradient of the non-zero component of the electric induction field must blow-up in finite time. Some growth estimates for solutions, which are valid on the maximal time-interval of existence are also derived.

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